Complexity Classes in Three-dimensional Gravitational Agents

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Abstract

Three-dimensional Gravitational Agents are presented as discrete dynamical systems with simple construction but showing ordered, chaotic and complex behaviors. This paper explores these systems searching for complexity classes and emerging patterns. It shows the existence of a phase transition in the space of their transition functions and it argues that the most complex pattern dynamics is located in the vicinity of this phase transition.

Introduction

In this paper we introduce and explore a class of discrete dynamical systems named three-dimensional (3D) Gravitational Agents (GA) based on a multi-agent model. The aim of this study is to show that this class of dynamical system exhibit a large spectrum of behaviors including ordered and chaotic dynamics, and that the most complex emerging patterns can be found in the vicinity of a phase transition between the ordered and chaotic phases. Such a study has already been performed on other classes of dynamical systems such as one- and two-dimensional Cellular Automata (Wolfram 1984; Langton 1991; Heudin 1996; Magnier & Heudin 1997). This research is part of a project addressing the "evolution of complexity" in various classes of dynamical systems (Heudin 1998).

First, we introduce the 3DGA model more formally. Then, we turn to a qualitative overview of its dynamics given a parameterization of the 3DGA space. Evidence is presented that these dynamics fall into a small set of distinct classes. We show the existence of a phase transition in the space of 3DGA rules and we argue that we can locate the most complex pattern dynamics in the vicinity of this phase transition. Finally we discuss these results and their possible relationships with cosmological simulations and observations, and biological evolution.

Three-dimensional Gravitational Agents

The 3DGA model Gravitational

Agents are defined as a model for a class of complex systems containing large numbers of identical or heterogeneous agents in which each agent interacts with all other agents in a 3D space. The state S_i of an agent i is completely specified by a floating point variable and two floating point vectors:

Let m_i be the "mass" of the agent specified as a double floating point value,

Let x_i be the 3D coordinates of the agent in the 3D space as double floating point values,

Let vx_i be the 3D velocities of the agent in the 3D space as double floating point values.

The state of an agent i evolves by iteration of the mapping:

$$S_i^{(t)} = F[S_0^{(t-1)}, S_1^{(t-1)}, \ldots, S_{i-1}^{(t-1)}, S_{i+1}^{(t-1)}, \ldots, S_N^{(t-1)}] \quad (1)$$

where N specifies the number of agents in the 3D space. F is the function specifying the transition rule which is described by the equation:

$$\frac{d_i^x}{dt^2} = G \sum_{j,j \neq i} \frac{m_j(x_j - x_i)}{(r_{ij}^2)^{\frac{3}{2}}} \quad (2)$$

where G is the gravitational constant and r_{ij} the distance between the agent i and the agent j in the 3D space. Note that this equation reflects the computer implementation of the transition rule.

The resulting transition function F constitutes a physics for a simple and discrete space-time universe. We theoretically consider an infinite 3D space even if we are interested in a small region of this space. The rationale is that this is the most realistic way to simulate the evolution of an agent distribution, since an artificial periodicity would lead to undesirable results due to boundary conditions.

Integration scheme

As agents approach each other, the forces get much bigger and the integration errors can get unacceptably large.

To avoid this, we used a 4^{th}-order Runge-Kutta integration scheme for computing both x_i and vx_i when considering a small number of agents (Garcia 1994). In addition we used a variable dt that cut the time step down when agents are near each other and increase it when agents are far away. For experiments with a large number of agents, we used a simple time-centered leap-frog integration scheme (Garcia 1994). The main reason is the high computational cost of a more complete Taylor series when considering large sets. The consequence is that we have reduced the rate at which the error accumulates by choosing small step size. Thus, for each experiment, dt was chosen in order to keep the global energy near constant, that is conserved to $\approx 1\%$.

Parameterizing the space of 3DGA rules

3DGA are characterized by two sets of parameters which influence the behavior of the resulting dynamical system. The first set concerns by the transition rule itself, while the second concerns initial conditions.

There are two parameters for the transition rule: G is the gravitational constant and dt defines the time step used by the integration. These two parameters define the set D of possible transition functions.

For the second set, we consider the initial conditions characterized by a configuration of N agents which are distributed in a cubic area of the infinite 3D space. When considering a random distribution, the initial density is given by the size D_{scale} of this cube and the number N of agents. The V_{scale} parameter determines the range of the initial velocity vectors, which are then set randomly using a Gaussian random process. In the case of homogeneous masses, all values are set to the M_{scale} parameter. In the case of heterogeneous masses, each agent gets a randomized mass value using M_{scale} as a range.

Imposing a parameterization scheme on the space of GA rules is quite straightforward and allows us to define a natural ordering of the rules. In this paper, we address the subspace of Δ characterized by a progression of the G parameter.

Implementation issues

The main problem with 3DGA is that it is highly computationally intensive due to the nature of the transition function. A simple experiment with only 10^3 agents requires up to 10^6 calls to a procedure for one update phase. Let assumes that this procedure includes 10 floating point operations and that we want to simulate 10^3 steps of interactions, this results in 10^{10} floating point operations. To reduce the amount of computation, many methods have been considered (Bertschinger & Gelb 1991). All the experiments reported in this paper were conducted using the direct summation approach because of its accuracy and flexibility.

The probability of a configuration leading to collisions is small in our experiments. However, in order to avoid divergence of the system at $r_{ij} = 0$, we implemented a "catching exception" approach. This solution was chosen rather than the classical ε softening parameter that adds an epsilon value to the distance between agents (Bertschinger & Gelb 1991). Thus the implementation is more accurate and is well suited for processing agents collisions in future works.

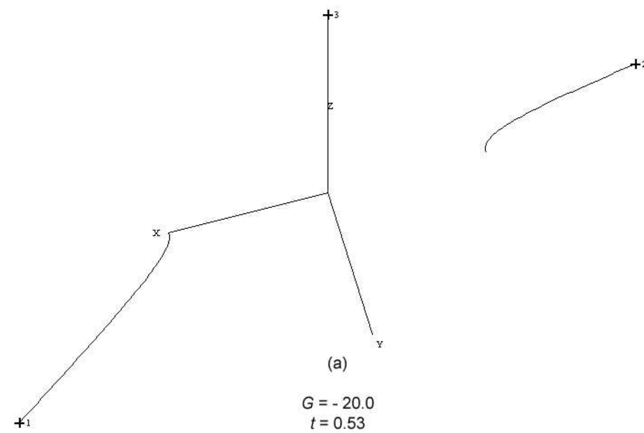

Figure 1: 3-agent example with $G < 0$

Qualitative Overview of 3D GA dynamics

Experiments with a small number of agents

In this section, we describe experiments using 3 or 5 agents with increasing G values. The initial conditions are inspired from (Diacu & Holmes 1996). We use the 4^{th}-order Runge-Kutta integration scheme with a maximum dt of 0.01. In most of these experiments, agents are initialized with the following states:

1. $m_1 = 1.0, x_1 = \{1, 0, 0\}, vx_1 = \{0, 1, 0\}$,
2. $m_2 = 1.0, x_2 = \{-1, 0, 0\}, vx_2 = \{0, -1, 0\}$,
3. $m_3 = 1.0, x_3 = \{0, 0, 1\}, vx_3 = \{0, 0, 0\}$.

With a negative gravitational constant (cf. figure 1), the distance between each agent increases regularly with time. The negative gravitation modifies the initial velocity conditions and results in an endless expanding dynamical behavior. Since this behavior is fully predictible with no change in time, it is interpreted as an ordered behavior.

With no gravitation ($G = 0.0$), each agent follows its initial velocity conditions. Figure 2 shows that agents 1 and 2 are moving straightforward in opposite directions while agent 3 remains in its initial position. This dynamical behavior is interpreted as a steady state since the system keeps its initial velocity conditions without any change.

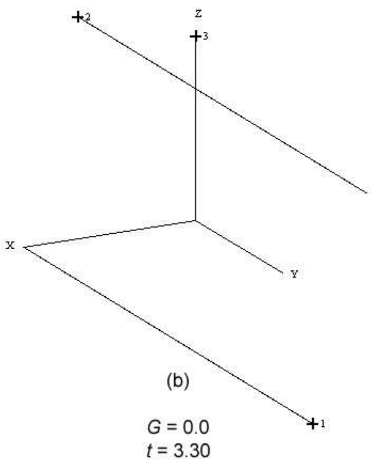

Figure 2: 3-agent example with $G = 0$

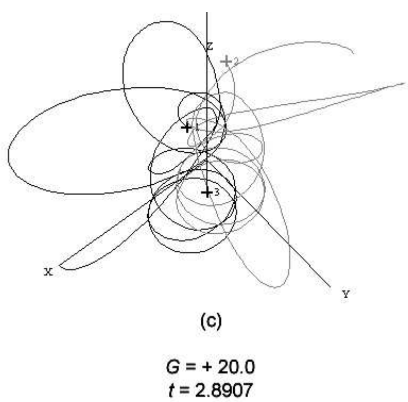

Figure 3: 3-agent example with $G > 0$

A positive gravitation induces a collapsing dynamical behavior which is combined with initial velocities of the three agents (cf. figure 3). Agent 1 and 2 show complex orbits. Agent 3 bounces up and down along the z axis in a chaotic way, as it picks up energy from the two other masses. This collapsing motion is interpreted as a chaotic dynamical behavior.

Figure 4 shows collapsing behaviors with 5 agents instead of 3. This experiment creates a system of two pairs of orbiting agents, with the fifth agent located on the z axis. When run, the two agent pairs orbit each other with unstable ellipses, while agent 5 is bounced back and forth between them chaotically.

When considering increasing values of the gravitational constant, the collapsing behavior becomes more violent. Note that an excessive increase in G generally results in an unstable system where the error accumulates in the first steps of the run, thus making agents shoot out very fast.

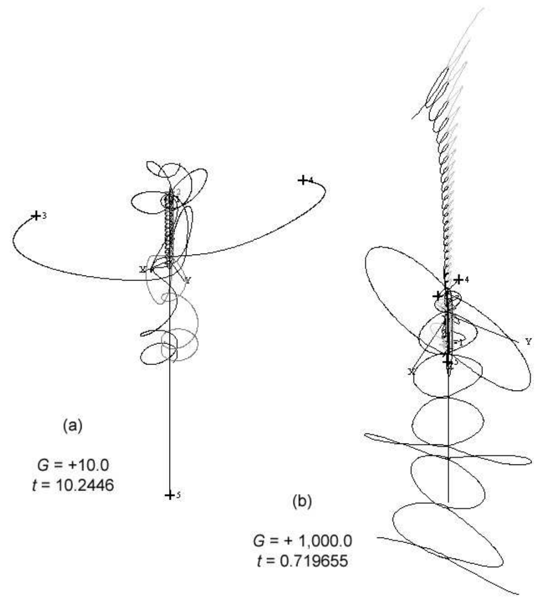

Figure 4: 5-agent experiment with $G = +10$ (a) and $G = +10^3$ (b)

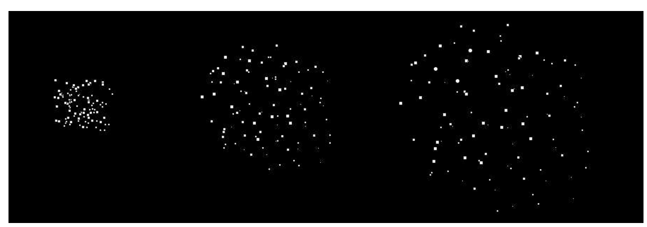

Figure 5: Expanding behavior with $G = -100$ for 100 heterogeneous agents ($t = 0, 10$ and 20)

Experiments with a large number of agents

The same kinds of experiments have been conducted with $N = 100$, 10^3, 10^4 and 10^5 agents with increasing G values. Even with this greater number of agents, the resulting rules are seen to generate the same qualitative behaviors. Patterns obtained with different randomized initial states differ in details, but exhibit the same global qualitative features.

Figure 5 shows an experiment involving 100 heterogeneous agents ($M_{\text{scale}} = 10^4$) with a negative gravitational constant. The agents cloud seems to explode with an overall speed which is proportional to the G value. This expanding behavior continues forming a quasi-sphere of agents with a decreasing density as the simulation goes on.

Figure 6 shows two typical patterns for small positive values of the gravitational constant. After a transient of several hundred time steps, a majority of agents forms a high density core with local chaotic dynamics, while some agents continue to move away from it. This agent cloud keeps its integrity and exhibits a chaotic regime

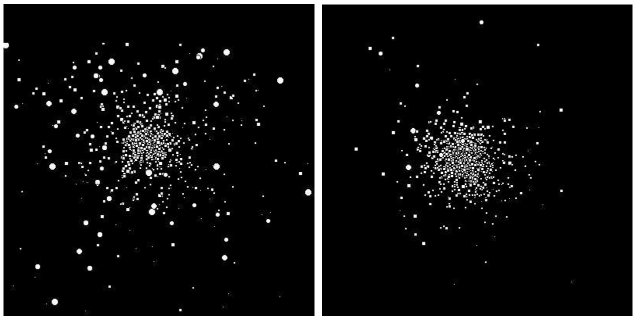

Figure 6: Collapsing behavior for 10^3 agents at $G = +10, t = 30,000$ (left) and $G = +0.1, t = 32,313$ (right)

around its gravity center which remains close to its initial position. Sometimes, in the case of a relatively low positive gravity, one can observe ephemeral collective behaviors like some agents orbiting around the center of gravity of the cloud in the same direction. These dynamical patterns disappear when a fluctuation of the agents' cloud is sufficient for breaking the movement.

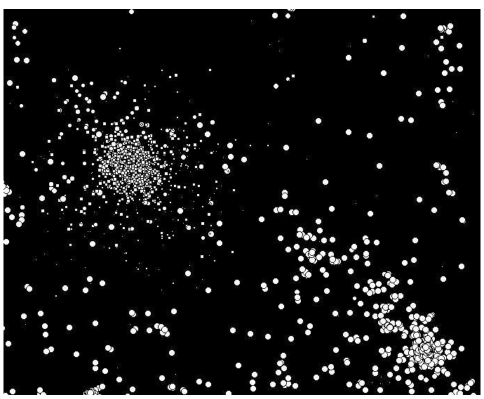

Figure 7: Collapsing behavior for 10^4 agents at $t = 200$

We report some experiments with $N = 10^4$ and 10^5. Again, we observe the same kinds of dynamical behaviors. Figures 7–8 shows two examples for $G = +10$. Figure 7 shows 10^4 agents grouped in one main core and a small set of aggregates, each of them composed of nearly 100 agents.

Figure 8 shows 10^5 agents distributed in four clusters and a main chaotic aggregate of agents in the center of the system. Note that in this case, we have relaxed the energy conservation constraint.

Ordering complexity classes and phase transition

In the framework of Cellular Automata (CA), Wolfram has shown that CA exhibit the full spectrum of dynamical behaviors (Wolfram 1983) and proposed a qualitative classification of their dynamics in four classes (Wolfram

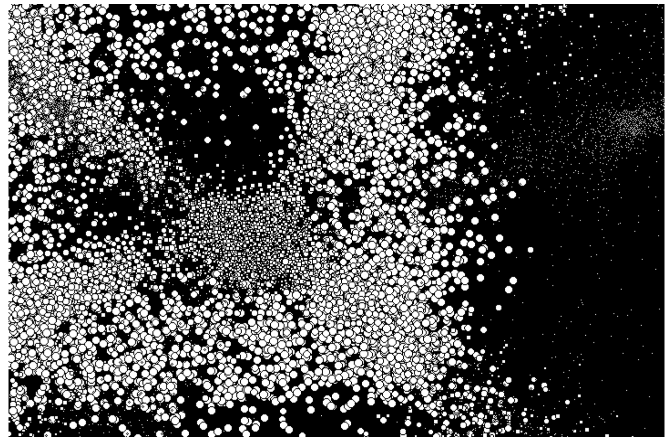

Figure 8: Collapsing behavior for 10^5 agents at $t = 81$

1984):

Class I is associated to limit points in the phase space.

Class II is associated to limit cycles.

Class III is associated to chaotic behaviors.

Class IV is associated to complex behaviors ; also some Class IV CA are suspected to support universal computation.

In 1991, Langton has proposed the "edge of chaos" hypothesis which claims the existence of a phase transition between ordered and chaotic behaviors for one-dimensional CA and locates complex dynamics in the vicinity of this transition (Langton 1991). Therefore, CAs with computational capabilities are likely to be found in the vicinity of the phase transition between ordered (Class I & II) and chaotic behaviors (Class III). More recently, evidence of this location have also been given in (Gutowitz & Langton 1995; Magnier & Heudin 1997; Packard 1998).

The experiments related in the previous sections of this paper give empirical evidence for the existence of three basic qualitative classes of dynamical behaviors for 3DGA:

- for $G < 0.0$, evolution leads to the "explosion" of the agents cloud,
- for $G = 0.0$, evolution follows the initial velocity conditions of the agents,
- for $G > 0.0$, evolution leads to the formation of a chaotic aggregation of agents.

There is a strong evidence that the value $G = 0.0$ forms a phase transition between ordered (expanding) behaviors and chaotic (collapsing) behaviors. We argue that complex patterns are located in the chaotic side

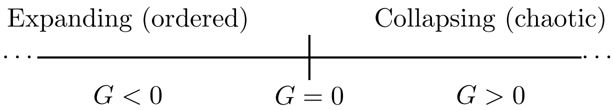

Figure 9: Ordering of the dynamical behavior classes as G increases

near this transition. This hypothesis is based on the fact that low values of the gravitational constant create long transients and enable the formation of more structured patterns while high values create violent behaviors which are not compatible with the emergence of these patterns.

While this classification cannot be strictly compared to the one proposed by Wolfram, we argue that 3DGA also exhibit the full spectrum of dynamical behaviors. As for CA, 3DGA configurations show limit points, limit cycles like orbiting agents and chaotic regimes. Also, emerging complex patterns in the form of agents aggregates can be located near the phase transition on its chaotic side. We suspect that the 3DGA model is able to support computation. A proof of this hypothesis remains to be done, for example by constructing "logical gates" using dedicated configurations of agents. However, it seems that the 3DGA model proposed in this paper cannot support universal computation since one cannot implement a structure which create an infinite and regular stream of agents, like the so-called "glider gun" of Conway's Life CA (Berlekamp, Conway, & Guy 1982).

Discussion

Relationships with cosmological simulations and observations The formation of galactic structures has traditionally been the domain of cosmological studies using Newtonian dynamics. Many simulations have been performed based on the so-called gravitational many-body problem — also called the N-body problem (Bertschinger & Gelb 1991). However, the focus has been on the simulation of observed features of our universe, such as interacting galaxies or large-scale patterns, using accurate models reflecting the reality "as-we-know-it". In this paper, we have rather studied the 3DGA model as an abstract class of dynamical system. It must be pointed out that one must add more features to get an accurate simulation of cosmological phenomena, like the expansion effect for example. However, the precision of our model is sufficient to compare our results to observations if we look for global patterns rather than detailed phenomena.

We can only compare results obtained with a positive G close to 0.0 since the value of the real gravitational constant is of $6.67259 \times 10^{-8}\text{cm}^3\text{g}^{-1}\text{s}^{-2}$. As expected, observed globular clusters and 3DGA patterns are characterized by surprising similarities, as shown by the two following examples.

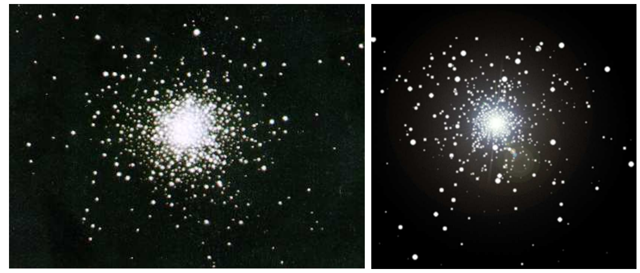

Figure 10: Globular cluster NGC1904 also called M79 (left), 3DGA with 10^3 agents (right)

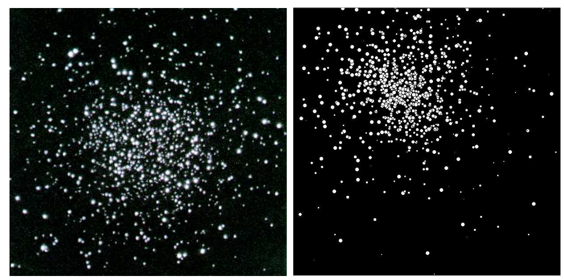

Figure 11: Globular cluster NGC5897 (left), 3DGA with 10^3 agents with $G = +1.0$ at $t = 32,500$ (right)

Figure 10 shows a real globular cluster NGC1904 (M79) on the left part, while the right part present a 3DGA composed of 10^3 agents with $G = +10.0$. This image is the same as in figure 6 (left), except that we have applied a simple rendering for adding light. Figure 11 shows a similar example with the NGC5897 globular cluster and a 3DGA composed of 10^3 heterogeneous agent with $G = +1.0$.

The fact is that some emerging structures obtained by 3DGA close to the phase transition are similar to observed patterns of our universe. This partially confirms our hypothesis about the location of complex patterns in the vicinity of the transition between the ordered and chaotic phases.

Relationships with biological evolution

One of the main speculative questions in Biology is the origin of life. Interestingly, many cosmologists are also speculating on the origin of the universe. This leads to the idea that, in both cases, the same principles of evolving complexity might be at work. While we are convinced that life is self-organized, might evolution and natural selection has some relationships with the creation of our universe?

This is not a new point of view. The theoretical physicist Lee Smolin was certainly the first to propose that natural selection might operate on the cosmic scale (Smolin 1997). Smolin's speculations were supported by many scientists from various fields, including Stuart Kauffman (1996). Smolin's theory is based on the idea that space and time can be modelized by a lattice structure on a tiny scale. In contrast, the 3DGA model attempts to study the evolution of complexity on a larger scale using Newtonian dynamics and an agent-based implementation. These two approaches are complementary by addressing a new research path to the study of life and complexity on the cosmic scale.

However, a lot of works remain to be done. Our first results are encouraging but some points need to be thoroughly addressed. First, an important difference compared to previous works on CA complexity classes (Langton 1991; Heudin 1996; Magnier & Heudin 1997) is that the phase transition is smooth rather than critical. This must be studied with both quantitative and qualitative experiments.

Secondly, the presented 3DGA model seems unable to support universal computation. This problem leads us to the idea that this model is too simple for showing emerging complex dynamics other than globular clusters (such as black holes, pulsar, etc). Thus, we plan to modify the model by including agents with matter aggregation and emission capabilities. We must also investigate other transition functions based on gravitational relativity and quantum theory. It is also probable that we need to run simulations with many more agents ($> 10^6$). However, these new features will not so difficult to integrate thanks to the flexibility and scalability of the agent-based approach.

Conclusion

In this paper, we have first introduced the three-dimensional (3D) Gravitational Agents (GA) model as a class of discrete dynamical systems using an agent-based approach. Then, we have presented a qualitative overview of 3DGA dynamics given a progression of the gravitational parameter. Evidence have been presented that these dynamics fall into three qualitative classes: steady, expanding, and collapsing, with a phase transition between ordered and chaotic behaviors. Then, we have argued that we can locate complex patterns in the vicinity of this phase transition. Future works include the study of the computational capabilities of the 3DGA model with more sophisticated agents, and the implementation of a fast hierarchical transition function based on the Barnes-Hut algorithm (Barnes & Hut 1996) in order to address simulations with more than 10^6 agents.

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