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Formal Definition of Self-reproductive Systems

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Abstract

Formal definition of self-reproduction may have importance the Alife research program, especially for application of its achievements outside the discipline. The paper examines two formal definitions of self-reproduction, suggested by McMullin and Löfgren. It is pointed out that these definitions form two major branches of self-reproduction analysis, described by ancestor-progeny and system-environment relationship. The ancestor-progeny definition allows to distinct between the exact / inexact reproduction. The system-environment definition brings in the original classification also allowing to differentiate between major classes of self-reproducers.

Introduction

A phenomenon of self-reproduction has an ultimate character, at least for our part of the Universe. Although presently studies focus around technical and biological applications, social systems are also an important case of self-reproducers (Luksha 2002).

Formalization of what is a self-reproducing system is substantially important for research programs in Alife (as set out by Langton (1989)), especially in a sense that such formalization provides basis for classification of self-reproducers. The latter is important for model design, since different types of self-reproducers may employ different techniques and strategies to reproduce themselves.

It should also be emphasized that artificial life models may well be transplanted back to natural and humanity sciences (by which they were first inspired) in order to understand better the phenomenon of self-reproduction, primarily self-reproduction of biological and social systems. Accordingly, formal definition may be important here as well.

Two main branches found in literature can be generalized as "progeny-ascendant relationship" and "system-environment relationship" definitions.

Progeny-ascendant relationship

A formal definition of a self-reproducing system, proposed by Barry McMullin (2000). Let $s \in \Psi$ be a system[1] s in system class Ψ, and $O(s) \subset \Psi$ is a set of systems that system s is capable of constructing (O as an offspring). System s is capable of producing another system, if $O(s) \neq \emptyset$. Then, if $s \in O(s)$, s is self-reproducing.

One may possibly find flaws in this definition (especially the fact that networks of interdependently producing systems are not covered by this definition, e.g. DNA-RNA-enzyme synthesis), in fact, this issue is done away through axiomatization of 'systemhood' (or crude consideration of system boundaries as given).

The more considerable problem is that in order to reveal whether a given system is a self-reproducer, one must define class Ψ for which this is determined. If Ψ is defined as any material object, the definition describes any repeated process: e.g. an oscillation in wave-like processes, even in mechanic waves, shall be "self-reproduction". To avoid problem of self-reproduction non-triviality (Langton 1984), there are two possible ways of varying the definition. Either one puts phenomenological restriction to Ψ: e.g. only objects of engineering (machines), biology (living organisms) and social sciences (societies and institutions) can be self-reproductive. Alternatively, one restricts the minimal level of complexity of objects in Ψ (but then the issue of complexity measure comes into view, which may exclude intuitively 'proper' objects or may include intuitively 'wrong' objects). In any case, a concealed requirement is that an observer must exist that shall determine the content of class Ψ. While this should not represent a problem for the purpose of Alife model transplantation into other sciences, it may somewhat undermine a theory's 'objectivity' (should Alife researchers be attached to observer-independent positivistic paradigm).

The basis for classification of parent-progeny relationship is viewed as following. Some measure of qualitative difference, $d(l_i, l_j)$, can be introduced, so that:

- $d(l_i, l_i) = 0$ (a function has its minimum for an exact copy of a self-reproducer l_i).

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[1] While McMullin talks of *machines*, and his main issue is to find a definition for designable artificial life, I believe his definition is good enough to be generalized to a class of systems capable of producing other systems

type of new system production	criterion	description
exact replication	$d(s_0, s_t) = 0$ (or $d(s_t, s_{t+1}) = 0$)	difference between each new copy and the original system must be minimal
near replication of an ancestor	$d(s_0, s_t) < D$	each new copy imitates the original system, with possible reversible mutations
near replication of a parent	$d(s_t, s_{t+1}) < D$	a new copy must have resemblance with its parent, but not necessary with all its ancestors (and thus this is a process of irreversible mutations)

Table 1: Types of self-reproduction

- $d(l_i, l_j) \leq D$ if system is considered an imitation of a given one, where D is a level of acceptable variation (see Eigen *et al.*, (1981) for measures of this kind used in pre-life models).

The typology of ancestor-progeny relationship is analogue to Sipper distinction between self-replication and self-reproduction where copy being exact and inexact replica (Sipper *et al.* 1997). Three possible types of reproduction (exact replication, near replication of an ancestor, and near replication of a parent) are presented in Table 1, s_t being a system s produced in t-th generation. The case of near replication of a parent appears to be the most distributed naturally (and also initially studied by von Neumann (Aspray & Burks 1987)), although other cases may also exist[2].

A self-reproducing system, accordingly, is a system capable to produce its copies or imitations (which is, other self-reproducing systems with the equivalent, or similar, structure and functions), and it is a system created by another self-reproducing system with the equivalent, or similar, structure and functions.

System-environment relationship

All natural self-reproducers are purely material structures. Therefore, they must have matter and energy interactions with external environment, and they can only be reproduced through such interactions.

Following closely an approach proposed by Löfgren (1972), a refined definition can be suggested to describe a system reproduced in a given environment. A producing system S' urges its environment F to produce another system S'', by applying some “effort” (or targeted action) E to it:

$$(S' \xrightarrow{A} E) \rightarrow S'' \quad (1)$$

If S'' is such that S' and S'' have a substantial degree of similarity, then A is a process of self-reproduction. It is possible to say also that S' and S'' both belong to a system type S, and the definition can be written as

$$(S \xrightarrow{A} E) \rightarrow S \quad (2)$$

The action A transforms raw material of environment E into a target system S, also producing some non-usable by-product W. Then, it is possible to represent a process of self-reproduction in a from of an auto-catalytic reaction:

$$E + S \rightarrow 2S + W \quad (3)$$

S is self-reproducing in the environment of E, gradually “consuming” E in this process[3].

W denotes degraded matter and energy produced in the reaction which is not usable for further utilization by S. W may be usable for utilization by other self-reproducer types, or E may be renewable, so this process does not necessarily lead to the ‘heat death’.

It obvious that various types of systems self-reproductive in their given environment have a completely different physical structure and also a different complexity of organization and functioning (compare e.g. a computer virus to a reproduction of multi-cellar organism); also a complexity of their environment can be different.

It is possible to distinguish between types of natural reproducers depending on a degree of complexity of self-reproducer S (of complexity $c(S)$) in relation to its environment E (of complexity $c(E)$), as presented in Table 2. One of appropriate measures to compare qualitatively different classes of self-reproducers with substantially discriminate environment is the measure of quantity and variety of elements and links in systems considered, and the quantity and variety of operation types for such systems (Edmonds 1999).

Comparative complexity is not the only issue in self-reproduction. For each of these types of self-reproductive

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[2] A classification more specifically describing types of ‘near replication of a parent’ has been suggested by E. Szathmáry, classification based on hereditary potential and ‘mode of synthesis’ (Szathmáry 1995).

[3] Some self-reproducers, such as computer viruses or memes, can be thought of as reproduced at no cost, although a cost may be quite low so it can be neglected (energy required to reproduce a series of electronic signals is insignificant, especially when compared with amounts of energy required for hardware self-maintenance).

structures, there obviously exists a lower limit of complexity that would allow them to operate purposefully and in particular to self-reproduce. There are clear evidences from cellular biology that such a limit exists for biological self-reproducers, such as prokaryotic cells. A minimum structure of a cell must have 15%-20% of components of *E.Coli* (Watson 1976).

Bacteria (such as *E. Coli*) are quite a complex structures capable of self-reproducing in a mixture of rather basic organic molecules. Yet, a computer virus is a comparatively simple program which requires quite a complicated hardware and software to get executed (i.e. to self-reproduce). This may imply that there exists a lower limit of complexity for "system and environment" aggregate structure, allowing a system to self-reproduce in a given environment.

Structuring of self-reproduction studies can further be achieved through the given definitions and classifications. It is evidential that many models claimed to be universal (e.g. von Neumann's automaton) actually suit for a sub-class of self-reproductive systems (called "true self-reproducers" here). The distinction between various classes of self-reproducers may lead to models which on one hand suit the Alife research program being matter-independent (against what has been demanded in Emmeche (1992)) and yet become more specific by considering certain properties of the environment in which given systems reproduce themselves.

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$c(E)$ to $c(S)$	type of self-reproduction	description	examples
$>$	quasi-self-reproducers	strictly dependent in reproduction on a system of higher complexity not produced as a part of its reproduction process	• viruses and genes; • memes; • computer viruses and computer "artificial life" (e.g. Tierra (Ray 2001))
$\cong$	semi-self-reproducers	autonomous complex systems requiring another comparably complex system to self-reproduce	• organisms with sexual divergence; • (certain) organisms with parasitic reproduction
$<$	true self-reproducers	complex autonomous systems capable to self-reproduce in an environment of basic elements[a]	• prokaryotic / eukaryotic cells; • organisms with asexual reproduction; • self-reproducing society; • artificial self-reproducing plants (e.g. (Freitas & Gilbreath 1980))

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[a] From theory point of view, it has been a type of system modeled by von Neumann (1966). For biological systems, this case of self-reproducers has been described by Maturana & Varela (1980), because complex structures must be produced inside such systems out of basic environment.

Table 2: Typology of natural self-reproducers