# Self-reproducers use contrapuntal means

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## Abstract

By evolving self-reproducers, found in two-dimensional Cellular Automata rules, we obtain fractal-like objects. This paper focuses on analyzing the fractal features of different self-reproducers and provides a perspective on how it is possible to translate into music some creative processes self-reproducers show, which are similar to the counterpoint method. Many self-reproducers are tied together and combined to imitate or respond to the rhythm of other self-reproducers, thus realizing the inner beauty of the life-like- artificial shapes.

## Introduction

Since B. Mandelbrot (1975) introduced a new class of mathematical objects and a new branch of mathematical science, a growing number of composers have shown interest in the musical application of fractals (Hsu K. J. et al. 1990), (Hsu K. J. et al. 1991), (Pickover C. 1992) in the realm of the simulation of life-like arts into other digital media (Bilotta, Pantano and Talarico, 2000; Bilotta and Pantano, 2001; Bilotta, Miranda, Pantano and Todd, 2002). In this paper, the environment considered to obtain music is made up by two-dimensional CA rules, which contain self-reproducers (Bilotta, Lafusa, Pantano, 2002). The behavior of the self-reproducers, as we found in CA rules, is fractal-like. Using statistical methods, we have analyzed how fractal dimension of two and three dimensional shapes helps obtain different kinds of music. We have presumed that self-replication and self-similarity may be consistent characteristics of natural and artificial systems and of music as well. In this paper, we try to make these features available, in order to translate them into music and to ascertain if these traits may, in turn, develop creative musical compositions.

## Self-reproducers, Fractals and Music

The behavior of the self-reproducers we found in CA rules is fractal-like. Mandelbrot defined the fractal geometry, which offers many novel ways to describe and quantitatively characterize objects and events in nature, assigning a fractal dimension to them. A significant feature of fractals, which can be changed into music, is their self-similarity. How is it possible to detect these features and change them into music? If we start a simulation of evolved rules, after a short transient period, the self-replicating patterns emerge. The pattern in Figure 1 at time $T = 0$ produces spatial-temporal patterns, with a square-like symmetry. If we analyze the spatial configuration after 42 temporal steps (see Figure 2), it is possible to note the typical outline of the scaling invariance. To demonstrate that the shapes in Figure 2 have a fractal dimension, we have adopted a standard method, which let us activate a counting box in order to define the D parameter. This parameter describes how structural features are distributed at large and small scales within the object. The method let us analyze fractal phenomena using representation of two-dimensional (Figures 2 and 3) and three-dimensional (Figure 4) objects. With two dimensional objects like the ones in Figure 2, we get a fractal dimension $D \simeq 0,97$. This values is lower than 1 and lesser than the dust of the Cantor set. If we use the 2d representation of the spatial temporal pattern, we obtain the fractal dimension $D \simeq 1,54$.

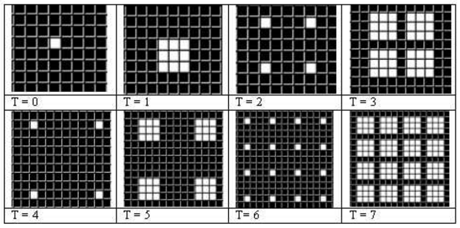

Figure 1: Evolution of a simple self-reproducing structure (1) in different time steps.

We have realized a three-dimensional image of this self-reproducer, which is shown in Figure 4.

We can translate these objects using different types of

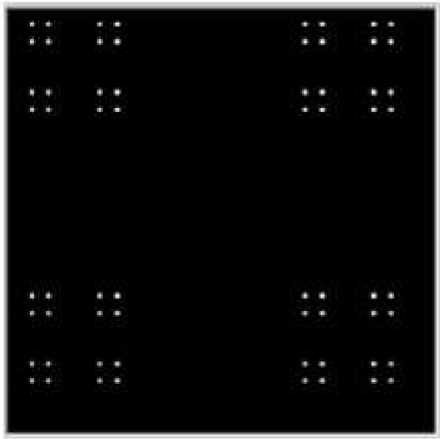

Figure 2: Scaling invariance of the self-reproducer (1) after 42 time-steps.

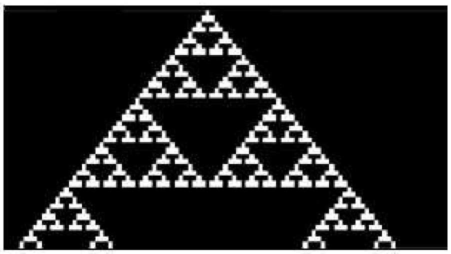

Figure 3: 2d shape of the self-reproducer (1).

codes that we have already developed (Bilotta and Pantano, 2001) to realize harmony. Harmony is produced when several pitches are used together. We think that these objects can contain repeated patterns, chords built on steps of the scale and can create a variety of textures that make up harmony. It is possible to create a melody as well, which is composed of pitches that may move higher or lower or don't vary, according to the repetition of patterns, to their variations, to their states. The evolution of the pattern gives the rise and fall of the pitch creating a melody with a distinctive shape or direction, which in turn follows the presence or the absence of a specific state. As for the spatial pattern, pitches may be organized in patterns, which may be combined to create phrases and melodies. Pitches may also be organized in specific ascending and descending patterns called scales. In other words, it is possible to realize music using the self-reproducers patterns.

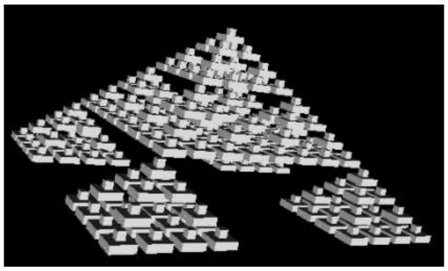

Figure 4: 3d shape of the self-reproducer (1).

In particular, from one of the patterns of the self replicator

$$\begin{pmatrix} 0 & 2 & 2 & 2 & 0 \\ 2 & 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 2 \\ 0 & 2 & 2 & 2 & 0 \end{pmatrix} \tag{1}$$

reported in Figure 5, emerging from a k4r1 CA, it is possible to calculate the number of the populations (the CA states) and associate to each line of the graph several pitches (see Figure 6). The pitches are obtained by dividing the lines into 40 intervals, considering the maximum and the minimum of the curves and associating to each interval a pitch of the 5 central octaves. In Figure 7, a transcription of the graph illustrated in Figure 6 is shown. The same kind of musification can be realized using the input-entropy function, as reported in Figure 8. The musical transcription in scores is presented in Figure 9. Many kinds of musical pieces can be realized composing the themes obtained utilizing the CA populations and the input entropy function, as the scores realized in Figure 10, and exploiting the narrative musical framework we have described in a recent paper (Bilotta and Pantano, 2002).

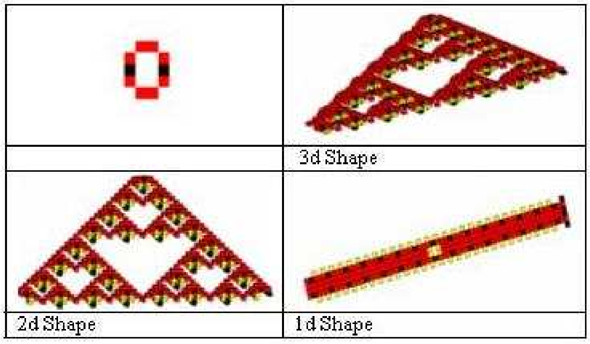

Figure 5: Different views of the self-replicator (1)

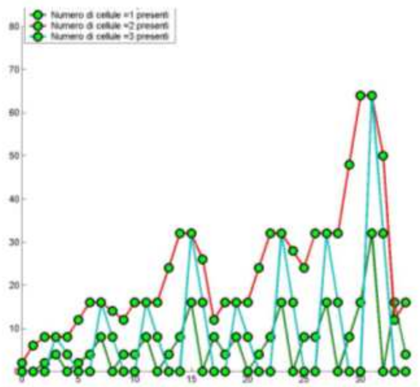

Figure 6: Graph of the self-replicator (1) populations, taking into account a spatial-temporal pattern of the self-replicator.
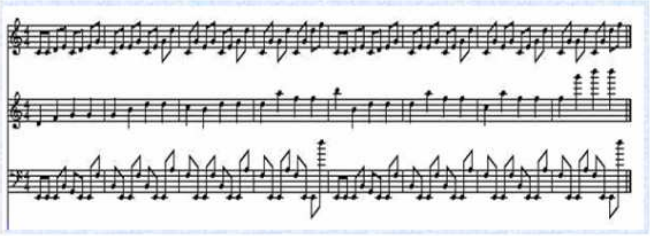
Figure 7: Musical transcription of the CA states, of one of the self-reproducer spatial temporal pattern.

## Do self-reproducers have rhythms of evolution?

If we make a self-reproducer evolve for a relevant number of steps, we see that it describes in time the curve reported in Figure 11. As the graphic shows, the increasing of the self-reproducer follows a constant tendency repeated over time, with changes in the replication scale (number of replications for each block given by the pattern, typical of the curve $9, 9, 15, 27, 15, 33, 9$. Such a replication occurs at different time scales (different times) and with different size scales (number of different replication) (see Table 1). This numerical scheme can be considered the rhythm of the self-reproducer evolution. Rhythm is the fundamental source of motion in music. Music evolves in time in longer and shorter durations that are often grouped together in patterns of sound and silence. The rhythm in music may vary from extremely pronounced to barely perceptible. This is determined by the tempo or speed of the beat. Meter is the organization and measurement of beats that are arranged in groups. These groups may be composed of two beats, three beats, or multiples or combinations of them. In fact, also this sequence is realized by three building blocks, made up of seven elements each, that we call **A**, **B** and **C**.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 9 | 9 | 15 | 9 | 27 | 15 | 33 | 9 |
| 27 | 27 | 45 | 15 | 45 | 33 | 63 | 9 |
| 27 | 27 | 45 | 27 | 81 | 45 | 99 | 15 |

Table 1: Numerical scheme of the self-reproducer (111).

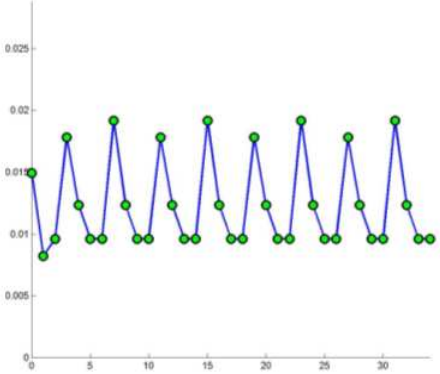

Figure 8: Input entropy of one of the spatial temporal patterns of the self-reproducer.
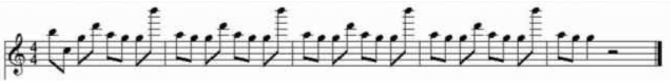
Figure 9: Theme realized by the input entropy function.

| **A** | | | | | | |
|---|---|---|---|---|---|---|
| 9 | 9 | 15 | 9 | 27 | 15 | 33 |
| **B** | | | | | | |
| 27 | 27 | 45 | 15 | 45 | 33 | 63 |
| **C** | | | | | | |
| 27 | 27 | 45 | 27 | 81 | 45 | 99 |

**A**, **B** and **C** have some interesting properties since they are made by other blocks. The first block can be subdivided in two blocks, one made up of 3 elements that we call

| **A1** | | |
|---|---|---|
| 9 | 9 | 15 |

Another block of 4 elements, that we call

| **A2** | | | |
|---|---|---|---|
| 9 | 27 | 15 | 33 |

**B** can be obtained by composing **B1** and **B2**:

$$B1 = 3 \times A1$$

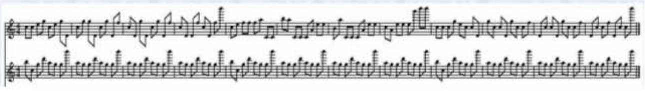

Figure 10: Musical scores realized putting together the themes obtained from the CA states and the input entropy function.

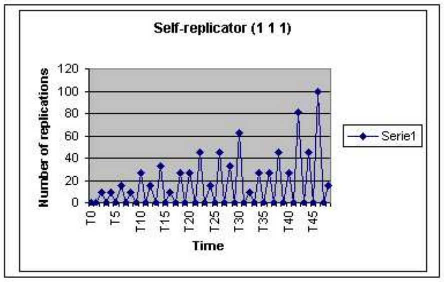

Figure 11: Graphic of the self-reproducing process of the structure formed by the states (1 1 1), obtained by a k4 r1 CA rules.

$$B2 = (9, A2 + 2 \times A1)$$

**C** is obtained directly from **A**

$$C = 3 \times A$$

Let us consider other 2 blocks, **S**, **T**.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 45 | 45 | 75 | 33 | 99 | 63 | 129 | 9 |
| 27 | 27 | 45 | 27 | 81 | 45 | 99 | 27 |

Each time the number which finishes the sequence of seven elements occurs, **A** is reproduced again.

$$S1 = 5 \times A1$$

The same does not happen for **S2**, which operates in a different manner

| | | | |
|---|---|---|---|
| 33 | 99 | 63 | 129 |

**T** is obtained directly from **A** and it is similar to **C**: $\mathbf{T} = 3 \times \mathbf{A}$.
We can say that this self-reproducer in its evolution uses three fundamentals and a variation .
Self-reproducer (1 1 1) explicates in its evolution the presence of another self-reproducer: (2 2 2), whose behavior in terms of number of replications is displayed by the graph in Figure 12. The sequence of its evolution is the following:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 5 | 9 | 11 | 9 | 15 | 15 | 21 | 9 |
| 15 | 27 | 33 | 15 | 25 | 33 | 43 | 9 |
| 15 | 27 | 33 | 27 | 45 | 45 | 63 | 15 |

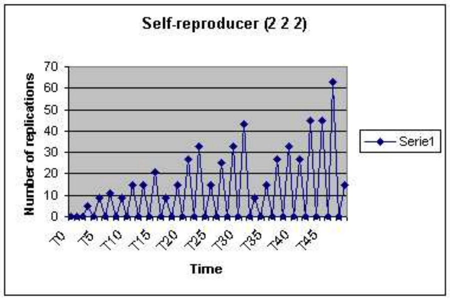

Figure 12: Evolution of the self-reproducer formed by the states (2 2 2).

In this self-reproducer there are 3 blocks: **D**, **E**, **F**.

| | | | | | | |
|---|---|---|---|---|---|---|
| **D** | | | | | | |
| 5 | 9 | 11 | 9 | 15 | 15 | 21 |
| **E** | | | | | | |
| 15 | 27 | 33 | 15 | 25 | 33 | 43 |
| **F** | | | | | | |
| 15 | 27 | 33 | 27 | 45 | 45 | 63 |

Each block is separated by three numbers $(9, 9, 15)$, as it happens for the other self-reproducer (1 1 1) and likely the **A1** block. It is possible to sub-divide **D** in **D1** and **D2**.

| | | |
|---|---|---|
| **D1** | | |
| 5 | 9 | 11 |

and

| | | | |
|---|---|---|---|
| **D2** | | | |
| 9 | 15 | 15 | 21 |

But, while **D2** is obtained following the rule: $D2 = (9, 6 + A1)$; **E1** as: $E1 = 3 * D1$; and $F = 3 * D$, **E2** varies.

| | | | |
|---|---|---|---|
| 15 | 25 | 33 | 43 |

The self-reproducer (2 2 2) is composed of two fundamentals and by a variation (a break of the symmetry). Otherwise, if we consider the presence of A1, manifested by the self-reproducer (1 1 1), we can say that this self-reproducer uses 3 fundamentals to evolve. Putting together the self-reproducers (1 1 1) and (2 2 2) we obtain the curve shown in Figure 13. If we translate into music the numerical sequences of the self-reproducers evolutions with the code used for the CA states, we can say that the self-reproducer behavior is similar to a canon or a fugue, which utilizes different voices or fundamentals. Let us introduce the concept of canon. According to the literature, a canon is a precise form of imitation of a voice (*dux, the leader*) by another voice (*comes, the follower*). Literally speaking, canon means a rule to be

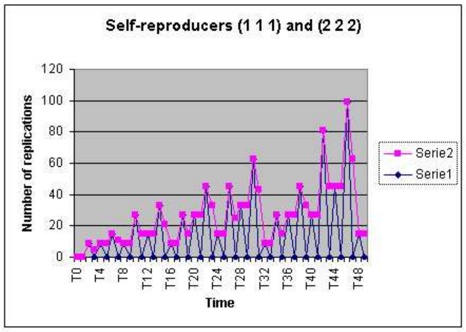

Figure 13: Evolution of the self-reproducers (1 1 1) and (2 2 2) considered together.

followed in order to obtain this imitation process.

Traditionally, in music there are many canons. In the *strict canon* the leader is given, while the follower repeats exactly the dux melodic line, according to a pre-fixed distance of entrance and eventually to precise intervals of connection (for example, of the octave, of the fifth and of the fourth intervals). At the end of the canon, the voices end one after the other or end together. In the *circular canon* the voices, instead of finishing, start again from the beginning, giving the idea that the canon can be continued *ad libitum (canon pertetuus)*. In the *twisted canon* the dux ends a tone over in regard to the beginning and, at every repetition, the voices arise of an octave. In the *enigmatic canon* there are neither known point of entrance nor relationships of intervals, which have to be detected. The *mixed canon* consists of a direct canon with some uncontrolled parts, or free variations.

The elements that characterize the canon are the number of voices, the distance of entrance, their intervals of connection, the direction of movements of the *comes.* In the *direct canon*, the *comes* respects the direction of the dux intervals. In the *inverted canon*, the *comes* advances realizing a movement in the opposite direction, as if the intervals were reflected in a mirror. Let us realize some of the described canons, using the numerical scheme of the self-reproducers above described. If we use the convention:

| | | |
|---|---|---|
| 5=D | 9=C | 11=F |

and change the notes within the chords intervals when we have multiples of the basic numbers, we obtain:

| | | | |
|---|---|---|---|
| 18=E | 27=G | 15=F | 33=A |

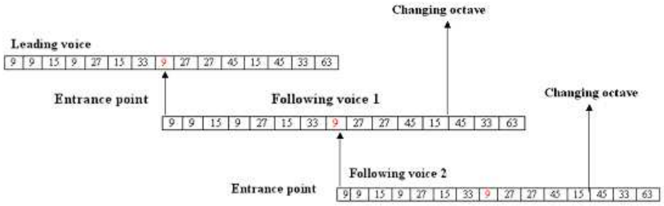

Figure 14: Schematic representation of the unison canon, with the entrance point and the changing of the octave.
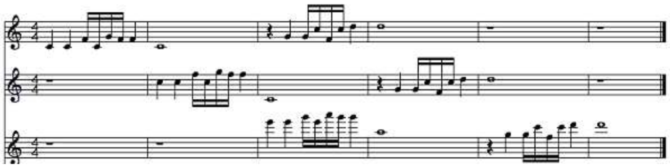
Figure 15: Unison canon

In this way we can realize the unison canon, as displayed in Figure 14. In a unison canon the follower performs precisely the same melody of the leader. As the name implies, canon at the octave involves repetition of the leader an octave higher or lower. A transcription of the scheme is reported in Figure 15. The same happens if we think to realize the *inverted canon*, whose schema and musical transcription are displayed in Figure 16 and 17. We think that also for the fugue it is possible to activate this narrative musical framework, since the availability of the self-reproducers let us produce many musical pieces. By setting up artificial musical universe of modular in-

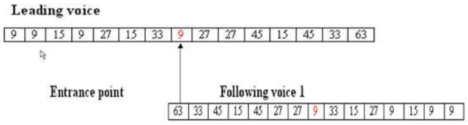

Figure 16: Scheme of the inverted canon.
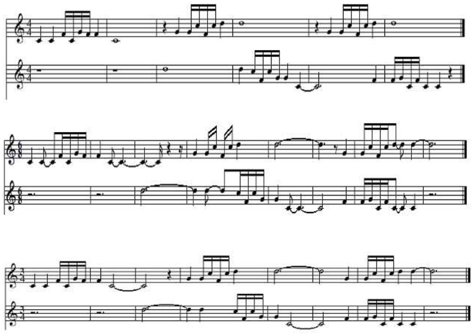
Figure 17: Three different time signature for the inverted canon

terlocking themes, derived by the self-reproducers behavior, and creating a set of rules by which they may be combined, musicians have the possibility to create truly creative compositions.

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