Towards a Definition of Dynamical Hierarchies

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Abstract

Hierarchical organisation is omnipresent in natural systems and is of special importance in connection with living systems. Thus far it is poorly understood what type of artificially created system is capable of replicating the emergent hierarchical structure of nature. This paper is an attempt to clarify what hierarchical structure could be.

Introduction

Probably nobody doubts that hierarchical organisation is both essential and omnipresent in the natural world, but upon pondering on the subject one quickly realises that the topic is actually not as straightforward as it might seem at first. As so often is the case in science, what seems clear to the intuition is hard to put into words. One major problem is certainly that "hierarchy" can mean many things.

For example, armies are hierarchical systems, in the sense that there are one-way command structures which have to be strictly obeyed. Scientific theories can be hierarchical. Quantum Mechanics encompasses Newton's classical mechanics, and a Grand Unified Theory will maybe sometimes encompass all fundamental forces of physics into a single theory. Also all organisms are parts of a hierarchic system: Our species *Homo Sapiens* is well placed within a well organised taxonomic system (Hominae, Hominidae, Hominoidea, Catharrhini, Anthropoidea, Primates, etc...).

All those examples of hierarchical systems are interesting and important, but of relatively little concern for the present article. Rather than systems which are internally differentiated according to some relative assignments of rank, the objects of concern here are composite entities. We take the word "composite" in this context just as importantly as the word "entity," i.e. we are not interested in mere assemblies of things, but rather in *individualised* entities which themselves are composed of and can be analysed into other individualised entities.

Most, if not all, natural systems are composite in this sense and we believe that the hierarchical structure is essential for the correct and efficient functioning of those systems; further we suggest that the complexity of the world as we see it is a result of its highly hierarchical structure. Astonishingly, the problem of dynamical hierarchies is often omitted in scientific models and in the past relatively little attention has been spent on its origins and properties.

Casti(Casti 1997) enthusiastically describes how modern computer technology can be used to create virtual copies of real systems, which enable us to make ever more accurate models of the real world. While we in principle share his optimism, we also think that, as far as computer modelling is concerned, the methodological development is out-paced by the technological development. Memory size and processing speed are often not the sole limiting factors for computer models any more; in more and more cases it is a lack of theoretical and methodological understanding that prevents us from implementing better models. The question about the definition of life, our inability to create open-ended evolutionary systems, or the question about the generation of novelty in computer models, to name but a few, are getting more and more vexing than the mere effectiveness of our computational media.

The understanding of the function and functioning of hierarchical organisation in nature and more importantly, the ability to simulate artificial worlds that display the emergence of comparable structures will be an important contribution to more realistic, powerful and interesting models.

In this paper we would like to give an initial discussion of the problems involved. Our focus is agent-based models of living systems, but many of our arguments and statements will equally apply to other agent-based models or even other types of models altogether. We do not pretend or claim to have solved any open problem, nor to provide a comprehensive review. The goal of this paper is to propose some threads for discussion and promote future work on the subject in the Alife community.

We will detail a sequence of increasingly more detailed definitions, each of which will capture another aspect of our intuitions about dynamical hierarchies and attempts to rectify shortcomings of the previous definitions. In this discussion we use the domain of monomers, polymers and micelles, following the notions discussed by Rasmussen(Rasmussen *et al.* 2001), to focus our ideas.

Hierarchical Systems

Chinese Characters

Before we attempt to find a definition of dynamical hierarchies, we would like to develop an intuition for what a dynamical hierarchy is and why they are important and useful. We start with an example.

The written Chinese language consists of many thousand different characters of which each stands for one one-syllable word. To the un-initiated those characters seem to be just symbols with arbitrary semantic contents, and correspondingly the task to learn them appears to be gigantic. This first impression could not be further from the truth. Chinese characters are highly structured composite pictograms which by their very structure tell the reader a great deal about their semantic contents and also in many cases about their pronunciation; the proficient reader is often able to guess the meaning of a previously unencountered character.

At least one reason why Chinese characters are so highly intuitive is that they are structured in a hierarchical manner. The basic, atomic, component of all characters is the so-called basic stroke, of which there are eight. Different spatial arrangements of the basic strokes form the second layer of the language, the so-called "Radicals," of which there are about 200. These spatial arrangements are often loosely inspired by the shape of the denoted object. Radicals can stand alone, corresponding to a specific one-syllable sound in a sentence, but are also used as parts of composite characters; as parts of more complicated characters, Radicals provide clues about meaning and pronunciation. The hierarchic structure does not stop here: Notions that are described by only one word in English are often expressed by a combination of two character symbols in Chinese; the meaning of the combination is then often only associatively related to the meaning of the participating characters.

The Chinese written language is thus clearly a hierarchical system, with different compositional principles at each layer. The second layer is formed by composing the primitive elements (the basic strokes) to simple pictograms; the representation is often highly abstract, but typically the result of a simplification process from more concrete historical ur-forms. At the next level, the level of the character, meaning and pronunciation guide the composition of the Radical to composite characters. Furthermore, at the level above, notions are often formed by two composite characters. At each level different compositional principles apply.

The hierarchical structure of the Chinese language makes it possible to keep the difficulty of acquiring the skill to read and write it at an acceptable level while the complexity of the writing system itself is for all practical purposes infinite; the complexity of the objects increases with each level. There are only a few basic strokes and nothing can be expressed by them; Radicals on the other hand would allow some limited communication. Using only composite characters one has already a fairly rich vocabulary, but everything can be expressed once one also masters strings of composite characters (notions). In this way it is possible to assemble increasingly more complex higher order structures from very simple basic building-blocks.

Evidently, Chinese characters are a cultural phenomenon and their hierarchical structure is imposed by an external agency and not a product of the self-directed development of the system itself; in this sense they are very different from the type of self-organised dynamical hierarchies one can find in natural systems.

Dynamical Hierarchies

Let us now turn the attention to natural hierarchical system: Life itself. It is hardly necessary to convince the reader that all organisms are composed of several hierarchical layers. It would go far beyond the scope of this article to give an extensive account of the hierarchical structure of organisms (see in this context (Scott 1995)).

Approximately halfway between the macroscopic realm of organisms and its tiniest constituent parts are the proteins, an important and powerful class of bioactive molecules. Essentially, they can be thought of as composed of less than two dozen amino acids. In the first instance linear, a protein will fold into a secondary and tertiary structure which defines both its spatial form and its biochemical properties. In organisms the functions of proteins are usually directed towards other macromolecules and include most importantly catalytic functions of high specificity but also various binding, splicing and cutting operations on DNA and RNA and proteins. In this sense one can say that the lower level interactions at the level of the amino-acids are of different kind than the interactions of the proteins themselves.

Like Chinese characters proteins are highly complex structures that consist of relatively few and simple building blocks; furthermore they are themselves building blocks of even more complex higher order structures. We recognise thus a similarity: The complexity of higher levels tends to be higher. However, note that there is a crucial difference: The complexity of Chinese characters and the structures they form is not self-organised, but imposed. No matter how much ink there is, no matter how many brushes there are, never will a Chinese poem be written unless there is a writer. Proteins do not necessarily need an external agency which constructs them: They are themselves the building blocks and the necessary components of the higher order entities (organisms) that ensure their ongoing production and maintenance. This causal closure is sometimes seen as a defining characteristic of life itself; the relevant keyword in this context is *autopoiesis* (Varela, Maturana, & Uribe 1974; Varela 1979), but also note the relevance of Robert Rosen's notion of complexity (Rosen 1991; 1999).

The hierarchical structure of proteins exceeds that of Chinese characters in another aspect: The hierarchy they form is dynamical, in the sense that they are as wholes dynamical entities but also in the sense that the interactions between their constituent parts are of dynamic nature. For example, if mixed together they will typically interact in a number of interesting ways, change their internal configurations in reaction to external stimuli, decay, unfold, re-fold, form new proteins or catalyse reactions or display any other dynamic behaviour. Proteins, can therefore be regarded as autonomously dynamic agents. Typically, this will not be true of Chinese characters, which always need an outside agency that interprets them and puts them into the right context, and will not be dynamic agents themselves.

Dynamical Hierarchies and Artificial Life

Organisms, by their very nature, are composed of interacting sub-parts. An indivisible atomic object, we would not call it living; the same is true for composite but inert and inactive objects. In the context of a field like Artificial Life it is therefore of crucial importance to understand how and under which conditions *dynamical* hierarchies spontaneously emerge from the interactions of computationally simulated first order structures only.

A natural method to investigate this question are agent-based models (ABM). Rather surprisingly, it turns out that it is relatively difficult to devise rules and agents which lead to the spontaneous emergence of a rich and interesting dynamically hierarchic structure. In most currently available ABMs there are only two hierarchical layers, namely the level of the primitive agents and the aggregate level, as for example the emergence of certain survival strategies (in evolutionary systems), pattern formations, predator-prey dynamics and the like. In many cases the aggregate level is not a composite dynamic agent, but often only a function of some observable properties of the primitive agents or a rather inert assembly of primitive agents. The latter case represents the trivial case of a dynamic hierarchy. Consider as an example the autopoietic structures in the so-called SCL-model (see Maturana and Varela (Varela, Maturana, & Uribe 1974) and Varela and McMullin (McMullin & Varela 1997); for an improvement see also McMullin and Groß(B. McMullin and D. Gross 2001)). This is a simple artificial chemistry consisting of a few types of particles which collectively realise a self-repairing and spatially well-defined (autopoietic) higher order entity. Unfortunately, this entity does not do much except growing until it decays. It can therefore hardly pass as an artificial living object.

A much more convincing example of an Artificial Life model with higher order dynamic entities is Rasmussen and coworkers' (Rasmussen & Smith 1994; Mayer & Rasmussen 1998; Rasmussen *et al.* 2001) attempt to demonstrate the spontaneous emergence of micelles from the bottom-up. In their model they are able to show how primitive agents (the "monomers") interact with one another to form polymers (claimed to be second order structures). Those polymers in turn assemble to micelles, which the authors claim to be third order structures. There has been some recent controversy concerning the exact interpretation of the hierarchical order of those micelles. If nothing else, this discussion highlighted a certain ambiguity in the definition of what defines a dynamical hierarchy.

Dynamical Hierarchies and Agent Based Models: Towards a Definition

Rasmussen's micelles certainly satisfy the intuition about what a third order structure should be. Obviously, intuition is not enough and it is indispensable to formulate a more rigorous criterion that specifies under which condition a composite object forms a new hierarchical layer. Such a criterion is at least necessary if one attempts to have a meaningful discourse about a general methodological *ansatz* for the generation of dynamical hierarchies in artificial systems; it is then essential to be able to compare the simulations between different frameworks. Rasmussen and coworkers are, of course, conscious about this and consequently do provide such a criterion. We will come back to this later.

In the following paragraphs we will attempt to develop an understanding of the difficulties and problems that are in the way of the formulation of a definition of dynamical hierarchies. We will hereby focus on artificial systems, especially on agent-based models, because we think that this is—at least as far as Artificial Life as a field is concerned—the most challenging and urgent problem. Our approach is to define a number of tentative definitions and to discuss their shortcomings. This will then give some insight into a few of the key-problems of any definition of dynamical hierarchies. We will conclude this section with a proposed definition of dynamical hierarchies in ABMs and similar formal systems.

Before we proceed a few words of caution are appropriate: Especially in the field of Artificial Life, there are a number of concepts which on the one hand seem to have a definite reality, but also successfully evade any attempts of definition. The most notorious examples are "Life" and "Emergence." Everybody agrees that there are living objects out there and that those objects are in some sense fundamentally different from non-living objects; the problem is only to formulate what exactly this difference is. Correspondingly, also everybody will agree that the world is hierarchically organised and that there are composite objects out there. However, it is not clear that there is a clear-cut demarcation between composite and primitive objects. Furthermore, one should be open to the possibility that the theoretical interests and points of view of the observer might have an influence on the perception of hierarchical structure. Ultimately, if true, this might mean that there simply is no general definition of hierarchy which everybody could agree on in general. We believe nevertheless, that, even if this is so, it is not only fruitful, but also necessary to attempt to find a working definition at least within particular contexts (here: ABMs).

Intuitively it is clear that objects can only form a higher order object if they interact with one another. Actually, it would be sufficient that the objects are connected to each other through chains of interactions, i.e. if A, B and C are objects, then A and C need not engage in an interaction as long as A and B and B and C do so. We say that A and C interact *indirectly*. We thus formulate our first tentative definition.

Tentative Definition 1 *If two or more objects, of which at least one is of order $N-1$, but none is of order higher than $N-1$, engage in indirect or direct interaction with one another, then they form an object of order N.*

The incompleteness of this definition becomes clear at a first glance. There are at least two components lacking from this first attempt: Firstly, we would demand from a hierarchical structure that it has some extended life-time. That is, a short, flash-like, interaction will not be sufficient to span a higher order object. We would therefore wish to add a temporal component to the definition in the sense that the interactions must last for a minimum period of time.

Leaving this aside, there is another serious problem. Not all types of interaction will really yield higher order objects, but only certain types of interactions. One could for example demand a minimum "strength" or intensity of the interaction; this will then of course quickly lead to the problem of finding an objectively and unambiguously applicable measure of strength of interaction. For the moment, however, this aspect need not worry us too much, since even if we had a clear-cut distinction between hierarchy-forming and non-hierarchy-forming interactions, this tentative definition leads to ambiguous and contradictory results. To see this imagine two first order objects A and B; we assume that A and B engage in an interaction which fulfils all of the criteria of strength and length in time; we can thus think the object AB to be a second order object. If we imagine another first order object C which enters into an interaction with AB and we also assume that this interaction meets the above criteria, then we would now according to the above definition be forced to consider the interacting pair AB and C as a new 3^{rd} order structure ABC.

Often this conclusion would be in striking contradiction to any intuitive idea of what a third order object is. Let us illustrate this using a simple example: If A and B are monomers and their interaction is bonding, then the new object, the polymer, is clearly of second order. The interaction "bonding" can thus be assumed to fulfil the yet un-formulated criteria. Note now, that if C is also a monomer and it bonds with B (and thus with AB), then ABC would fulfil the definitional requirements for a 3^{rd} order structure. However, intuitively it is clear that the resulting trimer ABC is at most of second order (as it is only a longer polymer, not a new structure). It seems therefore that the above tentative definition is not at all sufficient. We can thus conclude, that any definition of hierarchical objects based on interaction alone is not sufficient.

It seems that something else in addition to interaction is required in order to form a new hierarchical level. Intuitively, higher level objects must somehow distinguish themselves from mere collections of interacting lower level objects; one might require them to have a new property which cannot be found at lower levels. This is exactly the line of thought which Rasmussen and coworkers followed with their definition of hierarchies in ABMs based on Baas' notion of hyper-structure. Inspired by this approach we can formulate the following tentative definition:

Tentative Definition 2 *An object A is of order N if it is an assembly of directly or indirectly interacting lower order objects of which at least one is of order $N-1$ and if A has a new kind of property, that cannot be found at lower order objects.*

A problem with this definition is of course to understand what a new "kind of property" might be. We will not dwell on this too much, only highlight the intuition behind this formulation: Imagine two cars, one red one blue. They will both share the same kind of property "color," even if the value this property takes might be different. On the other hand, we can now imagine that the blue car has airbags, whereas the red car does not. The blue car might then be regarded as possessing a new type of property relative to the red car. Admittedly, this characterisation might become ambiguous when applied to specific cases, but for the purpose of the present discussion it will be sufficient to rely on the intuition.

To understand the scope and limitations of this definition, consider a toy artificial chemistry, consisting of only one type of agent; this agent moves around on a, say, square lattice grid according to some movement-rules. Each agent in the artificial chemistry can form bonds to at most two other agents in its immediate von Neumann neighborhood. They are thus capable of forming a chain of bonded agents which closes onto itself. This chain partitions the simulated space of the artificial chemistry in two parts, the inside and the outside. This structure thus possesses a new kind of property: $P = \{\text{Defines inside and outside}\}$. According to the above definition we can now view this closed chain as a second order object, as it consists of first order interacting objects (the bonding agents) which do not posses P.

Instead of looking at closed chains, let us now take a look at a simple bonded pair of agents (or two-mer). Intuitively we would expect this structure to be a second order object as well. Indeed we easily find properties that are present at the collective level, but not below, such as $P_1 = \{\text{Has a new diffusion constant } d_2\}$ or $P_2 = \{\text{Can assume two different relative spatial configurations}\}$. Primitive agents posses neither P_1 nor P_2. Following the definition we are thus forced to conclude that already a pair of bonded agents is a second order structure.

Similarly, we will conclude that three-mers are second order objects (they posses no new kind of property relative to two-mers). However, if we add another monomer to a three-mer, then we will obtain a four-mer, which is in the specific geometry we are considering the smallest polymer which can form a closed chain and thus posses the property $P = \{\text{Defines inside and outside}\}$. Accordingly, we are forced to conclude that a four-mer is a third order structure; however, above we have concluded that a closed chain is a second order structure. This reasoning seems to indicate that tentative definition 2 leads to inconsistent results. We are thus forced to reject it.

While this definition is not completely equivalent to Rasmussen's and Baas' definition of higher order structures, the two approaches above certainly do share important features. It is therefore probable that a thorough analysis of Rasmussen's and Baas' definition will uncover similar inconsistencies as we have found in tentative definition 2. This is especially so, because the main source of the problem seems to be the requirement of a new kind of property at the higher level; this is a central element of both Rasmussen's and Baas' definition and tentative definition 2.

Proteins as Dynamical Hierarchies

Tentative definitions 1 and 2 are clearly too inclusive. While the case for the first tentative definition seems to be hopeless, because even a better specification of what counts as an admissible interaction would not remove the inconsistencies, it seems that the second tentative definition might be improved by a better specification of the types of properties that are sufficient to form a higher order structure.

In order to better understand what the options are one might take a look at a concrete natural system, for example, proteins. The interesting physical and chemical properties of a protein are largely dependent on its secondary and tertiary structure, which is attained by the folding of a linear chain of amino acids. First this overall conformation allows the protein to enter into a number of (on this level) novel *interactions* (such as catalysis, DNA-splicing etc...). We observe thus the emergence of new types of interactions at the higher level. This is a recurrent theme across many different types of composite systems: Atoms can form bonds, electrons, protons or neutrons cannot; cells can move around in the world and take up and process food, whereas proteins cannot do that and so on.

There is another aspect to this: The world seems to get more complex with increasing hierarchical order. The low levels of elementary particles, electrons and atoms are the field of physics, commonly regarded as a simple science, whereas complex systems arise at the higher levels of societies, biological organisms, eco-systems and so on. This rather informal observation is quite consistent with another aspect: Not only is there a trend towards new interactions at higher hierarchical levels, but there is also a trends towards *more* interactions per object. Those observations seem to map to the above mentioned *leitmotivs*; we will comment more on this below. For the moment we let ourselves be inspired by this to formulate another tentative definition:

Tentative Definition 3 *An object A is of order N if it is an assembly of directly or indirectly interacting lower order objects of which at least one is of order $N-1$ and if A can engage in at least one new type of interaction in which objects of order $< N$ cannot engage.*

Note that this definition is a special case of tentative definition 2 with the further requirement that the new property is a new interaction. Also note that A does not actually have to engage in that new interaction, just has the capability to do so.

There are a few problems with tentative definition 3. First, there is the immediate problem of what qualifies as a type of interaction, and specifically what qualifies as a *new* type of interaction. While the intuition behind this idea seems to be reasonably clear, in specific cases, we would have to expect difficulties of demarcation. Even if we assume this problem to be solved, the proposed criterion would still be too exclusive. Take the above example of a toy artificial chemistry: The twomers would not qualify as a higher order structure, because they certainly do not show any interaction that is not already present at the lower level (they merely engage in an interaction that can be found at the primitive level). The same would be true for the closed chain of bonded monomers or the autopoietic structures in the above mentioned SCL model and in fact for most intuitive second order structures in ABMs. While it is true that the hierarchic structure of ABMs is generally poor, it would go too far to deny them second order structures. We thus conclude that tentative definition 3 is too strict.

Tentative Definition 4

All of the above proposed definitions were insufficient either because they were too inclusive or too exclusive and as such produced obviously unsatisfactory results. We would now like to propose a further tentative definition, which mixes some elements of the above approaches and as such yields better results. The drawback of the following definition is that its scope will be limited to a certain class of objects with well defined interaction interfaces. As such, we can expect it to offer useful guidelines for artificial systems such as ABMs, but it will be less applicable to natural systems.

Before we can formulate our next tentative definition, we have to prepare some notation. Let $A = \{I^A_{N_l}, O^A_{M_j}, S^A_K\}$ and $B = \{I^B_{N'_i}, O^B_{M'_k}, S^B_{K'}\}$ be two objects where each of them has a number of N_l (N'_i) input interfaces of type l (i) and M_j (M'_k) output interfaces of type j (k) respectively in addition to the inner states S. Altogether, A has $I^A = \sum_{l=0}^{l=Itypes} N_l$ input interactions, but only $Itypes < I^A$ types of input interactions. A and B can interact and form a compound object $AB = \{I^A_{N_l}, O^A_{M_j}, S^A_K, I^B_{N'_i}, O^B_{M'_k}, S^B_{K'}, I^E_{N''_g}, O^E_{M''_h}, S^E_{K''}\}$, where the last group with the superscript E stands for possible emergent interaction interfaces and inner states. Note that the compound object has at most $I^A + I^B + I^E - 1$ input interfaces and $O^A + O^B + O^E - 1$ output interfaces (interaction between A and B consumes at least one interaction interface).

In most ABMs there will typically be a considerable overlap between the input and the output interactions, sometimes to the degree that there is no distinction between the two. The above mentioned toy-artificial chemistry is a point in case. The only interaction the agents can engage in is bonding (each agent can have two interactions of the type "Bonding"). Conceptually this is a completely symmetrical interaction: Each agent will therefore only have one input and one output interaction, namely bonding. One might however also imagine an ABM with two types of agents (let's call this example henceforth "modified toy artificial chemistry"): Agent type A has only one property, it exerts an attracting force to which agents of type B respond. In this example A would then have an output interaction, but no input interaction, while B would only have one input interaction, but no output interaction. If we add the capability of bonding to both types of agents, then the agents A would have two types of output interaction and one type of input interaction (bonding), while the agents B would have two types of input interaction and one type of output interaction (bonding).

Let us now assume that A and B are some arbitrary types of agents and the index sets $\eta = \{$*set of all indices that label active types of input interactions between* A *and* $B\}$ and $\omega = \{$*set of all indices that label active types of output interactions between* A *and* $B\}$ label the interactions between A and B. Note that both η and ω might contain labels of interactions from A and B. We can then formulate another tentative definition of dynamical hierarchies:

Tentative Definition 4 *AB is an object of order N, if*

- *A, B are of order $< N$,*
- *A or B or both are of order $N - 1$*
- *AB has a property that cannot be found at lower order objects*
- *At least one element of η or ω does not label an interaction in which subcomponents of A or B are engaging.*

This definition is limited to the case of only two interaction structures, but it is easy to extend it to the more general case of N interaction particles. We leave this to future work.

The basic idea of this definition is that we only accept a new level to be emerged if a new type of interaction has been essential in forming this level. Note that we do not demand this interaction emerge *de novo* at this level, just that it has not been used to form lower levels. This definition can be illustrated by means of an example: Consider again the above described toy-artificial chemistry (only one type of agent which can move on a square lattice and bond to at most two other agents) and add (similar to the modified toy artificial chemistry) an additional attractive inter-particle force, "gravitation." If we start with a random collection of unbound particles then, provided the gravitation force is weak compared to the strength of the bonding, we will first observe the formation of chains of bonded particles. Those chains will then form the second order objects. Note that according to the tentative definition 4 two-mers, three-mers, four-mers and N-mers are all of order two only: A three-mer, consisting of a two-mer and a primitive particle, fulfils the first three conditions of the definition, but the sub-components are spliced together by a type of interaction (bonding) which is already used at lower levels of one of its components (the two-mer itself consists of two bonded primitive particles). The three-mer thus fails the fourth condition.

However, through random diffusion and the effect of gravitation, we can expect the mixture of different primitive and second order particles to aggregate into clusters. Those clusters will consist of first and second order objects and are as such candidates for third order objects. The clusters also have emergent properties (they have an inside and an outside) and they are formed by a type of interaction that is not used in lower levels (gravitation). We feel thus justified to regard those clusters as third order objects.

We do not think that this definition can be universally applied to all types of systems, which certainly is a limitation: It is only applicable if the objects can be described as automata with input and output interfaces and an inner state. This certainly is possible for most ABMs, but real-world examples quickly lead to difficulties. Let us for example think of a car, certainly a higher order structure. Let us now assume that its primitive structures are objects such as the screws, rubber, cables, pipes and hoses and the like. Those objects are assembled to higher order structures such as the engine, the transmission, the gear box, the tyres and so on. Eventually, we would get the car as a whole. It is not clear to us that tentative definition 4 can be meaningfully applied in this context.

We see at least two reasons for this shortcoming: Firstly, as mentioned above, the definition itself is not suitable for this kind of real world system which lacks well defined interaction interfaces. Secondly, and probably more important, there might simply be no unique way to analyse general natural systems into hierarchically related sub-systems. It may ultimately be acceptable that the analysis of a system into hierarchically related sub-systems may always be defined only with respect to a given set of interfaces/interactions, and that therefore for some purposes conclusions about the composition of the system may be different from those for other purposes.

Despite those limitations for the application of the definition to real systems, we believe that the proposed definition can provide some valuable guidelines for the evaluation and comparison of the hierarchical structure of ABMs. We also think that the proposed definition can be used in order to sharpen the intuition when thinking about problems in connection with dynamical hierarchies.

Complexity and Hierarchies

We have already at the outset of this paper mentioned that it has turned out to be rather difficult to devise ABMs that display the spontaneous emergence of hierarchical structure, or even dynamical agents of second order. There are many reasons for this difficulty, but an exhaustive discussion of this topic warrants a paper of its own and therefore has to be deferred to future work.

We may nevertheless indicate one important aspect of this problem: It seems that in many ABMs the agents' interaction interfaces are quickly "used up." The composite object then lacks the ability for any further interactions and becomes a dead structure. Seemingly following this intuition, Rasmussen and coworkers have conjectured that high hierarchical order can only be realised in ABMs if the complexity (the "object complexity" as they call it) of the primitive agents, and their interactions is sufficiently high.

A glance at our proposed definition can help the intuition in this case. Assume that we have an ABM with hierarchical structure, but *without emergent terms*, that is the (primitive or higher order) agents, A and B form a higher order object such that

$$AB = \{I^A_{N_l}, O^A_{M_j}, S^A_K, I^B_{N'_i}, O^B_{M'_k}, S^B_{K'}\}.$$

A casual glance at the existing literature suggests that this (or slightly more generalised aggregates involving more than two participants) is the typical form of an emergent hierarchic object in ABMs (although there are probably no fundamental limitations that preclude emergent interactions in ABMs in general).

Systems of which the parts exclusively form hierarchical structures of this kind evidently have a reduced capacity to generate higher order structures. The limiting factor is the number of the different types of interaction interfaces of the primitive agents. If A and B are primitive agents, then they cannot generate more than $\min(I^A + I^B, O^A + O^B) + 1$ hierarchical orders; the second term stems from the fact that the primitive agents already span one hierarchical layer—the primitive level. Typically this number will be reduced by significant overlapping of types of interactions between the primitive agents: A more realistic upper bound will therefore be

$$\min(I^A + I^B - \bigcap I^A I^B, O^A + O^B - \bigcap O^A O^B) + 1,$$

where $\bigcap O^A O^B$ denotes the number of overlap of types between the input and output interactions of A and B respectively. In the special case of A and B being the same type of particle we will therefore have an upper limit of $\min(I^A, O^A) + 1$.

In the above example of a toy artificial chemistry with "gravitation" (one type of agent, 2 bonds per agent and gravitational attraction between agents) the agents have each two input interfaces (bonding and gravitation) as well as two output interfaces (bonding and gravitation); we would therefore expect at most three hierarchical layers in this model. Similarly, in the modified toy artificial chemistry (two types of agents, agent type A attracts B but not vice versa and bonding between agents possible) we also find that there can be at most three hierarchical layers: As far as input interactions are concerned, A has only one of them (bonding), whereas B has two (bonding, gravitation); thus $I^A + I^B = 3$. However, there is an overlap of one because both have bonding as input. Altogether we have therefore $I^A + I^B - \bigcap I^A I^B = 2$. Similarly we see that $O^A + O^B - \bigcap O^A O^B = 2$. Thus we see that there are no more than three hierarchical levels in this model.

If we identify object complexity with the number of types of interaction interfaces in the model, then the relation between the number of types of interactions and the potential for hierarchical structure corresponds directly to Rasmussen and coworkers' conjecture that the object complexity of the primitive agents in an ABM and its capacity to produce higher order structures are intimately related. At least as long as one accepts tentative definition 4 it is clear that this conjecture is correct whenever the higher order objects have *no emergent interactions* (i.e. there are no terms $I^E_{N''_i}, O^E_{M''_k}, S^E_{K''}$).

We conjecture that a review of the literature will show that most ABMs are of precisely this type. It is certainly true that ABMs do exhibit complex behaviour on the higher levels (often referred to as "macroscopic behaviour"), the higher level *agents* (if there are any) will have a strictly lower object complexity than the lower level agents (i.e. fewer types of interactions); those two types of complexity, the object complexity of the agents and the complexity of the collective behaviour of the agents must be clearly distinguished. In the context of ABMs we will therefore not be surprised to find Rasmussen's conjecture to be true. New levels can only be formed if there are unused types of interactions left; those, however, decrease by at least one at each level. The maximum number of hierarchical levels is thus determined by the object complexity of the primitive agents.

Many natural systems on the other hand, especially biological systems, will have emergent interactions at higher levels. One example is (as mentioned above) proteins, which acquire additional types of interactions through their tertiary structure. This observation also coincides with the physicist's intuition, that fundamental entities (such as protons, electrons, atoms) tend to be simpler to describe than higher level objects (such as proteins or organisms). The latter are currently outside the scope of any mathematical theory. It therefore appears that object complexity actually does increase with hierarchical order in many natural systems.

The physicist's intuition clearly is in contradiction to Rasmussen's conjecture because it assumes the emergence of agents with high object complexity (f.e. organisms) from the interaction of agents with low object complexity (f.i. electrons, protons, neutrons). There are now two possibilities to look at this: Firstly, one assumes the physicist's intuition to be wrong and finds, that once a decent definition of object complexity is applied, fundamental particles will be found to be of higher complexity than organisms. Secondly, and we find this the more fruitful approach, one can accept that object complexity increases in many natural systems and ask oneself why this is not the case in most ABMs. This will then lead to a challenging methodological question in artificial life: How can we construct ABMs which show increasing object complexity at higher levels? Or very similar, how can we construct ABMs with higher level agents that do show emergent properties?

Conclusion: A Central Challenge to Artificial Life Modelling

We have already noted that there is currently no theoretical argument suggesting that we cannot construct ABMs which form hierarchical objects with emergent interactions; at the same time however, we are not aware of any convincing example of an ABM where interactions actually do emerge at higher levels. Hence the limited emergence of hierarchical orders in simple ABMs.

Developing a general *ansatz* for an open-ended generation of dynamical hierarchies in ABMs, will greatly enhance the possibilities for modelling evolutionary phenomena and eventually also increase our (operative) understanding of life itself. Dynamical hierarchies are therefore central to Artificial Life and to develop an understanding of the pre-conditions that lead to the spontaneous generation of hierarchical levels is a methodological challenge.

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